Math 115
Winter 2017
Lecture 12

① Simplify: $\dfrac{-48x^7y^{-5}}{4x^3y^4} = \dfrac{-48}{4}\cdot\dfrac{x^7}{x^3}\cdot\dfrac{1}{y^5}\cdot\dfrac{1}{y^4}$

$= \boxed{-\dfrac{12x^4}{y^9}}$

Not a monomial

$x^n \cdot x^n = x^{m+n}$

② Distribute & Simplify: $4x^3(2x^2-5x+1)+20x^4-4x^3$

$= 8x^5 - 20x^4 + 4x^3 + 20x^4 - 4x^3 = \boxed{8x^5}$

Monomial
Deg. 5
Coef. 8

③ foil & Simplify: $(3x^2-4)(2x^2+5)$

$= 6x^4 + 15x^2 - 8x^2 - 20$

$= \boxed{6x^4+7x^2-20}$

Trinomial
Deg. 4, L.C. = 6, Const. −20

Use Special products $(A+B)^2 = A^2 + 2AB + B^2$ to find

1) $(7x+5)^2 = (7x)^2 + 2(7x)(5) + (5)^2$

$= \boxed{49x^2 + 70x + 25}$

2) $(3x^2 + 4y^3)^2 = (3x^2)^2 + 2(3x^2)(4y^3) + (4y^3)^2$

$= \boxed{9x^4 + 24x^2y^3 + 16y^6}$

Deg. 6
L.C. 16

3) $(4x^2 + 5x)^2$

$= (4x^2)^2 + 2(4x^2)(5x) + (5x)^2$

$= \boxed{16x^4 + 40x^3 + 25x^2}$

Use Special product $(A-B)^2 = A^2 - 2AB + B^2$

① $(2x-3)^2 = (2x)^2 - 2(2x)(3) + (3)^2$

$= \boxed{4x^2 - 12x + 9}$

② $(5x^3 - 10y^5)^2 = (5x^3)^2 - 2(5x^3)(10y^5) + (10y^5)^2$

$= \boxed{25x^6 - 100x^3y^5 + 100y^{10}}$

③ $(7x^4 - 4x^3)^2$

$= (7x^4)^2 - 2(7x^4)(4x^3) + (4x^3)^2$

$= \boxed{49x^8 - 56x^7 + 16x^6}$

Use special product $(A+B)(A-B) = A^2 - B^2$ to find

① $(5x+3)(5x-3) = (5x)^2 - (3)^2 = \boxed{25x^2 - 9}$

② $(7x^3 - 2y^8)(7x^3 + 2y^8) = (7x^3)^2 - (2y^8)^2$

$= \boxed{49x^6 - 4y^{16}}$

③ $(10x^7 + 5x^3)(10x^7 - 5x^3)$

$= (10x^7)^2 - (5x^3)^2 = \boxed{100x^{14} - 25x^6}$

Divide

$$\frac{36x^6 - 16x^4 + 4x^2}{-4x^2}$$

$= \frac{36x^6}{-4x^2} - \frac{16x^4}{-4x^2} + \frac{4x^2}{-4x^2}$

$= \boxed{-9x^4 + 4x^2 - 1}$

$$\frac{2x^2 - 5x + 3}{x - 1}$$

$$\begin{array}{r} 2x - 3 \\ x-1 \overline{)\,2x^2 - 5x + 3} \\ -(2x^2 - 2x) \\ \hline -3x + 3 \\ -(-3x + 3) \\ \hline \text{Rem.} \to 0 \end{array}$$

$x \cdot 2x = 2x^2$

$x \cdot (-3) = -3x$

$\boxed{2x - 3}$

Divide

$$\frac{50x^7y^4 - 30x^5y^3 + 20x^4y^2}{5x^3y^2}$$

$$= \frac{50x^7y^4}{5x^3y^2} - \frac{30x^5y^3}{5x^3y^2} + \frac{20x^4y^2}{5x^3y^2}$$

$$= 10x^4y^2 - 6x^2y + 4x$$

$$\frac{4x^2 - 8}{2x + 3}$$

$2x + 3 \overline{)\, 4x^2 + 0x - 8}$

$2x$ | $2x$ | $= 4x^2$

$2x$ | -3 | $= -6x$

$$\begin{array}{r} 2x - 3 \\ 2x+3 \overline{)\, 4x^2 + 0x - 8} \\ -(4x^2 + 6x\ \ \ \ ) \\ \hline -6x - 8 \\ -(-6x - 9) \\ \hline 1 \end{array}$$

$$2x - 3 + \frac{1}{2x+3}$$

Divide:

$$\frac{6x^3 + 9x - 13x^2 - 2}{2x - 1}$$

$2x$ | $3x^2$ | $= 6x^3$

$2x$ | $-5x$ | $= -10x^2$

$2x$ | 2 | $= 4x$

$$\begin{array}{r} 3x^2 - 5x + 2 \\ 2x-1 \overline{)\, 6x^3 - 13x^2 + 9x - 2} \\ -(6x^3 - 3x^2\ \ \ \ \ \ ) \\ \hline -10x^2 + 9x - 2 \\ -(-10x^2 + 5x\ \ \ ) \\ \hline 4x - 2 \\ -(4x - 2) \\ \hline 0 \end{array}$$

$$3x^2 - 5x + 2$$

Divide:

$$\frac{6x^3+13x^2-6x-10}{3x+2}$$

$$3x+2 \;\overline{)\; 6x^3+13x^2-6x-10}$$

$$2x^2+3x-4$$

$$-(6x^3+4x^2)$$

$$9x^2-6x-10$$

$$-(9x^2+6x)$$

$$-12x-10$$

$$-(-12x-8)$$

$$-2$$

$3x \cdot 2x^2 = 6x^3$ ✓

$3x \cdot 3x = 9x^2$ ✓

$3x \cdot (-4) = -12x$ ✓

$$2x^2+3x-4+\frac{-2}{3x+2}$$

Ch. 5 : Factoring Polynomials

⇒ Rewrite Polynomials in product form whenever possible

① Factoring out GCF (Reverse of Distribution)

$$2x-16 = 2x-2\cdot 8$$
$$= 2(x-8)$$

$$25x^2+10x = 5\cdot 5\cdot x\cdot x+2\cdot 5\cdot x$$
$$= 5x(5x+2)$$

$7x(2x-3) + 5(2x-3)$

$= (2x-3)(7x + 5)$

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① $3x - 15$

$= 3(x - 5)$

GCF

② $10x^2y + 5xy^2$

$= 5xy(2x + y)$

GCF

③ $4x^2(3x+5) - 7x(3x+5) + 1(3x+5)$

$= (3x+5)(4x^2 - 7x + 1)$

GCF

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② Factor by Grouping (4 terms or more)

$2x^3 + 5x^2 + 8x + 20$

$= x^2(2x + 5) + 4(2x + 5)$

$= (2x + 5)(x^2 + 4)$

$3x^3 - 7x^2 + 6x - 14$

$= x^2(3x - 7) + 2(3x - 7) =$ $(3x-7)(x^2+2)$

$x^3 + 8x^2 - 5x - 40$

$= x^2(x+8) - 5(x+8)$

$= (x+8)(x^2-5)$

$2x^4 - 5x^3 + 20x^2 - 50x$

$= x(2x^3 - 5x^2 + 20x - 50)$

$x^2(2x-5) + 10(2x-5)$

$= x(2x-5)(x^2+10)$

Recap of Factoring

① Factor out GCF

② Factor by Grouping

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③ Factoring Trinomial $ax^2 + bx + c$

$2x^2 + 5x + 3 = 2x^2 + 2x + 3x + 3$

$P = 6$

$S = 5$

6: $1, 6$ or $2, 3$

$= 2x^2 + 2x + 3x + 3$

$= 2x(x+1) + 3(x+1)$

$= (x+1)(2x+3)$

$4x^2 + x - 3$

$P = -12$

$S = 1$

$-1, 12$

-12

$-2, 6$

$-3, 4$

$= 4x^2 - 3x + 4x - 3$

$= x(4x - 3) + 1(4x - 3)$

$= (4x - 3)(x + 1)$

$5x^2 - 2x - 3$

$P = -15$

$S = -2$

$1, -15$

-15

$3, -5$

$= 5x^2 - 5x + 3x - 3$

$= 5x(x - 1) + 3(x - 1)$

$= (x - 1)(5x + 3)$

$4x^2 + 7x + 5$

$P = 20$

$S = 7$

$1, 20$

$2, 10$

$4, 5$

20

Prime Expression

Not factorable

$6x^2 - 19x + 15$

$P = 90$

$S = -19$

$-1, -90$

$-2, -45$

$-3, -30$

$-5, -18$

$-6, -15$

$-9, -10$

90

$= 6x^2 - 9x - 10x + 15$

$= 3x(2x - 3) - 5(2x - 3)$

$= (2x - 3)(3x - 5)$

Factoring:

1) Factor out GCF

2) Factor by Grouping

3) Factoring Trinomials ax^2+bx+c

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4) Factor Special binomials

$A^2 + B^2 = \text{Prime}$

$A^2 - B^2 = (A+B)(A-B)$

ex: $x^2+25 = x^2+5^2$ Prime

$x^2-36 = x^2-6^2 = \boxed{(x+6)(x-6)}$

$4x^2+9 = (2x)^2+3^2$ Prime

$9x^2-25 = (3x)^2-(5)^2 = \boxed{(3x+5)(3x-5)}$

1) $16x^2 + 49 = (4x)^2 + 7^2 =$ Prime

2) $25x^2 - 81y^2 = (5x)^2 - (9y)^2$

$= (5x + 9y)(5x - 9y)$

3) $x^4 - 16$

$= (x^2)^2 - (4)^2 = (x^2 + 4)(x^2 - 4)$

$= (x^2+4)(x+2)(x-2)$

Factor

1) $x^3 - 25x = x(x^2 - 25)$

$= x(x^2 - 5^2)$

$= x(x+5)(x-5)$

2) $x^3 + 5x^2 - 4x - 20$

$= x^2(x+5) - 4(x+5)$

$= (x+5)(x^2 - 4)$

$= (x+5)(x+2)(x-2)$

3) $x^2(x+3) + 6x(x+3) + 9(x+3)$

$= (x+3)(x^2 + 6x + 9)$

$P = 9$, $S = 6$

$1, 9$

$3, 3$

$= (x+3)(x+3)(x+3)$

$= (x+3)^3$

$x^2 + 3x + 3x + 9$

$= x(x+3) + 3(x+3)$

$= (x+3)(x+3)$

4) $4x^2(2x-5) - 25(2x-5)$

$= (2x-5)(4x^2 - 25)$

$\hookrightarrow (2x)^2 - (5)^2$

$= (2x+5)(2x-5)$

$= (2x-5)(2x+5)(2x-5)$

$= \boxed{(2x-5)^2(2x+5)}$

$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$

$$x^3 + 8 =$$

$$x^3 + 2^3 = (x + 2)(x^2 - 2x + 4)$$

$$x^3 + 125 =$$

$$x^3 + 5^3 = (x + 5)(x^2 - 5x + 25)$$

$$27x^3 + 1000 =$$

$$(3x)^3 + (10)^3 = (3x + 10)(9x^2 - 30x + 100)$$

$$64x^3 + 27y^3 =$$

$$(4x)^3 + (3y)^3 = (4x + 3y)(16x^2 - 12xy + 9y^2)$$

$$A^3 - B^3 = (A-B)(A^2 + AB + B^2)$$

$x^3 - 27 =$

$x^3 - 3^3 = (x-3)(x^2 + 3x + 9)$

$125x^3 - 8y^3 =$

$(5x)^3 - (2y)^3 = (5x-2y)(25x^2 + 10xy + 4y^2)$

I need 50L of 24% alcohol soln.
I have unlimited supply of 15% & 30% alcohol soln. How many liters of each?
Use system of linear eqns

15%	+	30%	=	24%
x		y		50

$$\begin{cases} x + y = 50 \\ \frac{15}{100}x + \frac{30}{100}y = \frac{24}{100}\cdot 50 \end{cases} \quad \cdot 100$$

$$\begin{cases} x + y = 50 \\ 15x + 30y = 24 \cdot 50 \end{cases} \quad \div 5$$

$$\begin{cases} x + y = 50 \\ 3x + 6y = 240 \end{cases} \quad \div 3$$

$$-1\begin{cases} x + y = 50 \\ x + 2y = 80 \end{cases}$$

$y = 30$

30L of 30%
20L of 15%

John needs 20L of 62.5% acid Soln.
He has Supply of 10% & 80% acid Soln.
How many liters of each?

10%	+	80%	=	62.5%
x		y		20

$$\begin{cases} x + y = 20 \\ \frac{10}{100}x + \frac{80}{100}y = \frac{62.5}{100}\cdot 20 \end{cases} \xrightarrow[\cdot 100]{} \begin{cases} x + y = 20 \\ 10x + 80y = 62.5(20) \end{cases}$$

$$\begin{cases} x + y = 20 \\ x + 8y = 62.5(2) \end{cases} \xrightarrow{-1} \begin{cases} x + y = 20 \\ x + 8y = 125 \end{cases}$$

$$7y = 105$$

$$y = \frac{105}{7}$$

$$y = 15$$

15L of 80% Soln
&
5L of 10% Soln.

Simple Interest

$$I = Prt$$

Find amount of interest of depositing $2400 for 1 year @ 4% APR.

$$I = 2400 \cdot \frac{4}{100} \cdot 1 \qquad I = 96$$

$96

Lisa deposited $2000 in two accounts. After 1 Year, She earned $139 in Simple interest. One account paid 5%, another one 8%. How much per account?

5% + 8% =

x y

$$\begin{cases} x + y = 2000 \\ \frac{5}{100}x + \frac{8}{100}y = 139 \end{cases} \quad \times 100$$

$$\begin{cases} x + y = 2000 \\ 5x + 8y = 13900 \end{cases} \quad \times -5$$

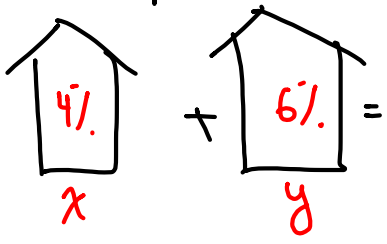
$$3y = 3900$$

$y = 1300$

$1300 @ 8%

$700 @ 5%

Lisa made $126 in total interest in one year. She invested some money @ 4% Simple interest and $300 less than twice that amount in 6% Simple interest. Find how much per account?



$$100\begin{cases}\frac{4}{100}\cdot x+\frac{6}{100}\cdot y=126\\ y=2x-300\end{cases}$$

$$\div 2\begin{cases}4x+6y=12600\\ y=2x-300\end{cases}$$

$$\begin{cases}2x+3y=6300\\ y=2x-300\end{cases}$$

$2x+3(2x-300)=6300$

$2x+6x-900=6300$

$8x=7200$

$x=900$

$900 @ 4\%$

$1500 @ 6\%$

Class QZ

① Divide: $\dfrac{52x^4y^3-32x^2y^5}{-4x^2y^3}$

② Divide: $\dfrac{4x^2+4x-30}{2x+7}$

SG 14

Due Wednesday

Cont. work on WP 9 & 10